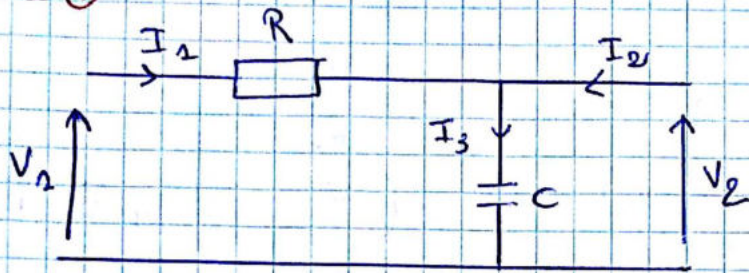


# Physique : Electronique Analogique

Serie: N°1

## Exercice (1)

1)



$$Z_c = \frac{1}{j\omega C}$$

ona

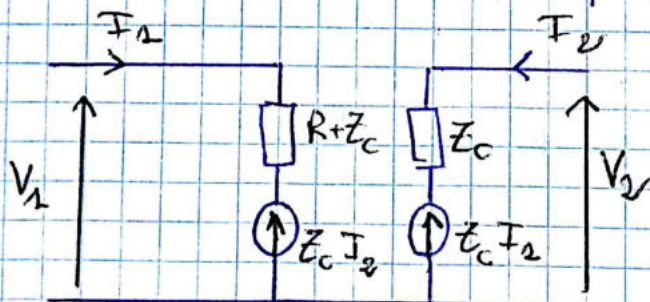
$$V_1 = R I_1 + Z_c I_3 = (R + Z_c) I_1 + Z_c I_2$$

$$V_2 = Z_c I_3 = Z_c I_1 + Z_c I_2$$

$$[\bar{Z}] = \begin{pmatrix} R + \frac{1}{j\omega C} & \frac{1}{j\omega C} \\ \frac{1}{j\omega C} & \frac{1}{j\omega C} \end{pmatrix}$$

ona  $Z_{12} = Z_{21}$  donc le quadripole passif.

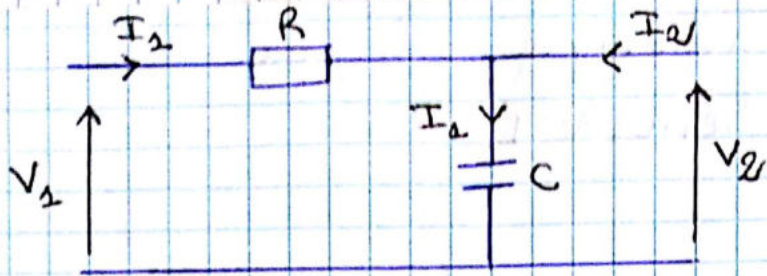
le schéma équivalent: (le modèle électrique correspondant pour la matrice impédance.)



$Z_c I_2$ : source de tension dépendante.

$Z_c I_1$ : source de tension dépendante.

2



$$V_1 = R I_1 + V_2 \Rightarrow I_1 = \frac{V_1}{R} - \frac{V_2}{R}$$

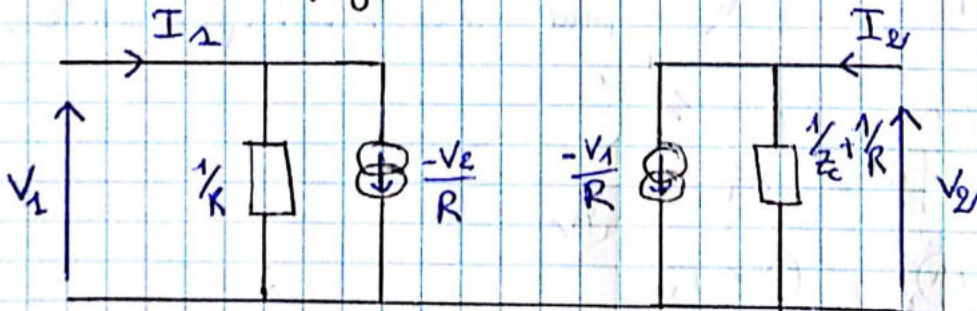
$$V_2 = Z_C (I_1 + I_2) = Z_C \left( \frac{V_1}{R} - \frac{V_2}{R} \right) + Z_C I_2$$

$$Z_C I_2 = V_2 - Z_C \left( \frac{V_1}{R} - \frac{V_2}{R} \right)$$

$$I_2 = \frac{V_2}{Z_C} - \frac{V_1}{R} + \frac{V_2}{R} = -\frac{V_1}{R} + \left( \frac{1}{Z_C} + \frac{1}{R} \right) V_2$$

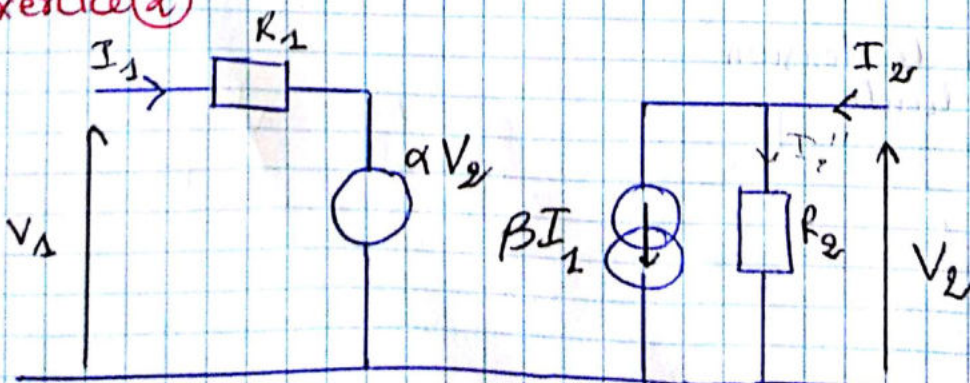
$$[Y] = \begin{pmatrix} \frac{1}{R} & -\frac{1}{R} \\ -\frac{1}{R} & \frac{1}{R} + j\omega C \end{pmatrix}$$

Le modèle électrique correspondant pour la matrice admettance  $Y_{ij}$ .



$-\frac{V_1}{R}$ ,  $-\frac{V_2}{R}$  sont des sources de courant dépendantes

Exercice 2



Il s'agit d'un  $\hat{Q}$  utilisant le modèle hybride dans ce cas, on exprime  $V_1$  et  $I_2$  en fonction de  $V_2$  et  $I_1$

$$\left\{ \begin{array}{l} V_1 = R_1 I_1 + \alpha V_2 \\ I_2 = \beta I_1 + \frac{1}{R_2} V_2 \end{array} \right. \text{ on sait que } \left\{ \begin{array}{l} V_1 = h_{11} I_1 + h_{12} V_2 \\ I_2 = h_{21} I_1 + h_{22} V_2 \end{array} \right.$$

•  $h_{11} = \left. \frac{V_1}{I_1} \right|_{V_2=0} = R_1$  : impédance

•  $h_{21} = \left. \frac{I_2}{I_1} \right|_{V_2=0} = \beta$  : le gain de courant (nombre)

•  $h_{12} = \left. \frac{V_1}{V_2} \right|_{I_1=0} = \alpha$  gain de tension (nombre)

•  $h_{22} = \left. \frac{I_2}{V_2} \right|_{I_1=0} = \frac{1}{R_2}$  : admittance

### Exercice ③

1)  $\begin{pmatrix} V_2 \\ I_2 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_1 \\ -I_1 \end{pmatrix}$

$$\left\{ \begin{array}{l} V_2 = A V_1 - B I_1 \\ I_2 = C V_1 - D I_1 \end{array} \right.$$

$A = \left. \frac{V_2}{V_1} \right|_{I_1=0}$  : gain de tension

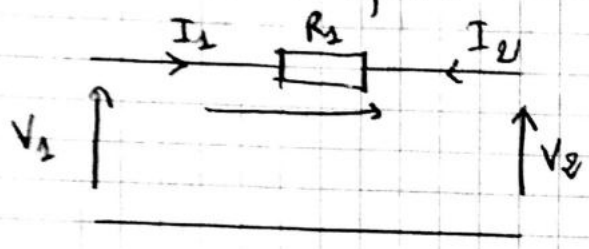
$C = \left. \frac{I_2}{V_1} \right|_{I_1=0}$  : admittance de transfert

$B = \left. -\frac{V_2}{I_1} \right|_{V_1=0}$  : impédance de transfert

$D = \left. -\frac{I_2}{I_1} \right|_{V_1=0}$  : gain de courant.

(a)

③ Matrice de Transfert:

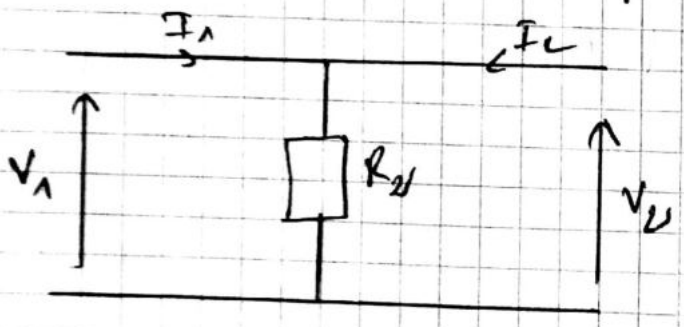


$$\begin{cases} V_1 = R_1 I_1 + V_2 \\ I_2 = -I_1 \end{cases}$$

$$\Rightarrow \begin{cases} V_2 = V_1 - R_1 I_1 \\ I_2 = -I_1 \end{cases}$$

Donc

$$T_1 = \begin{pmatrix} 1 & R_1 \\ 0 & 1 \end{pmatrix}$$



$$\begin{cases} V_2 = V_1 \\ I_3 = I_1 + I_2 \end{cases}$$

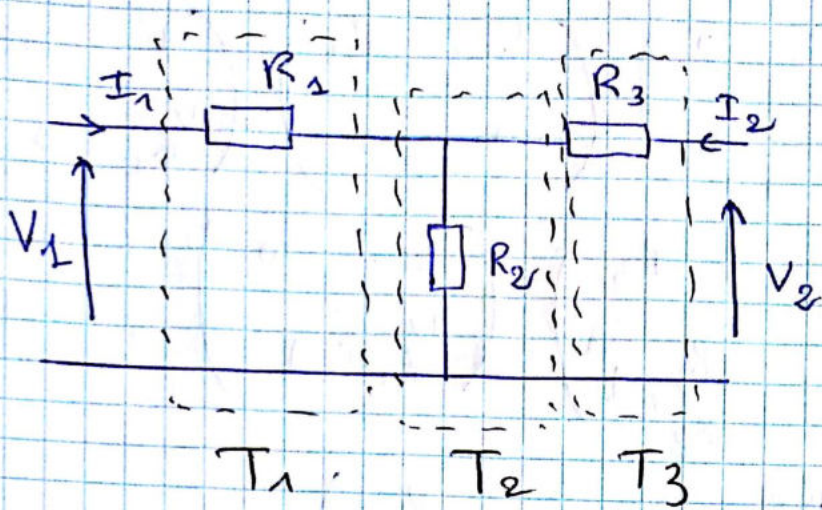
$$\Rightarrow \begin{cases} V_2 = V_1 \\ I_2 = I_3 - I_1 \end{cases}$$

ou  $R_2 I_3 = V_1 \Rightarrow I_3 = \frac{1}{R_2} V_1$

donc

$$\begin{cases} V_2 = V_1 \\ I_2 = \frac{V_1}{R_2} - I_1 \end{cases} \Rightarrow T_2 = \begin{pmatrix} 1 & 0 \\ \frac{1}{R_2} & 1 \end{pmatrix}$$

4)

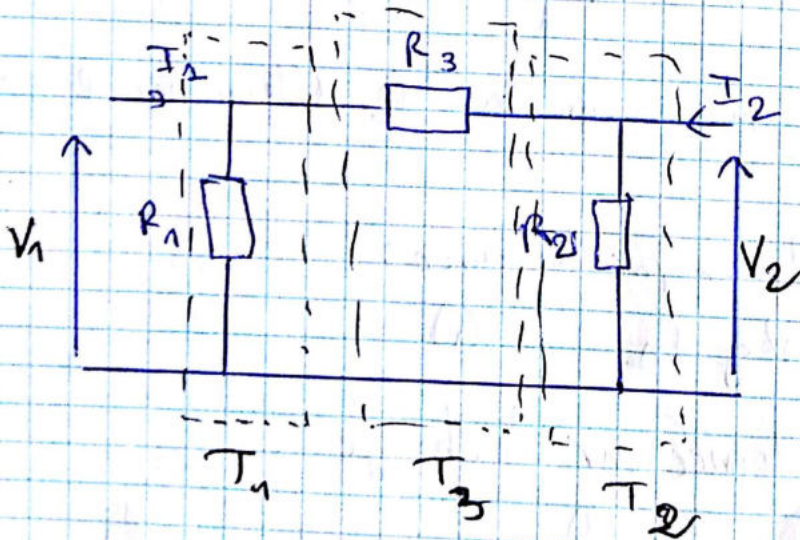


$$T_{eq} = T_3 \times T_2 \times T_1$$

$$= \begin{pmatrix} 1 & R_3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{R_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & R_1 \\ 0 & 1 \end{pmatrix}$$

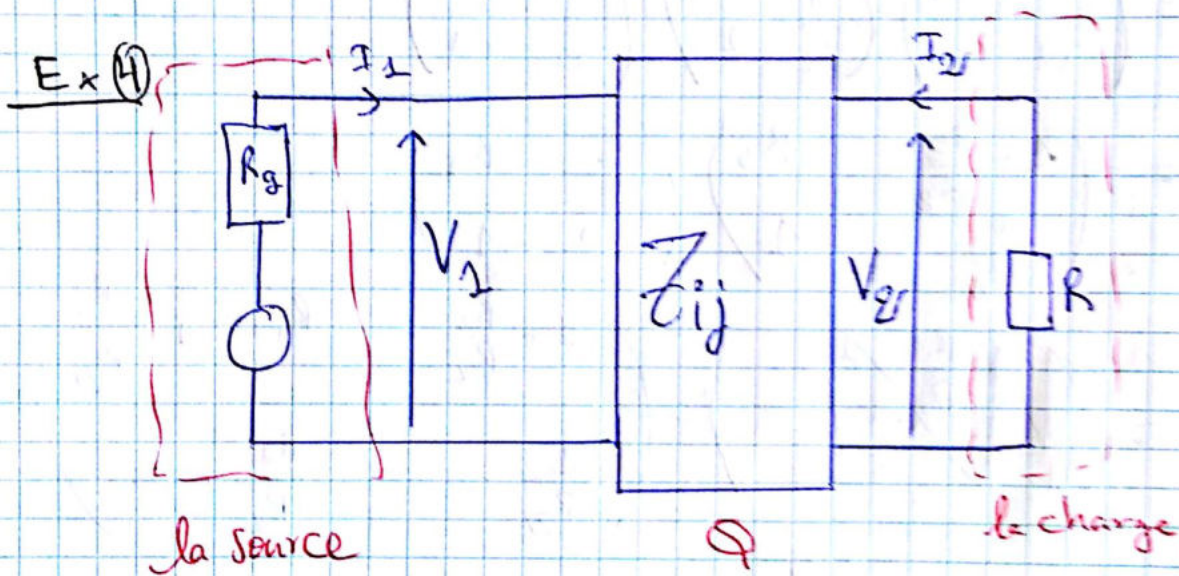
$$= \begin{pmatrix} 1 + \frac{R_3}{R_2} & R_3 \\ \frac{1}{R_2} & 1 \end{pmatrix} \begin{pmatrix} 1 & R_1 \\ 0 & 1 \end{pmatrix}$$

$$= \left( \text{Circuit Diagram} \right) \begin{pmatrix} 1 + \frac{R_3}{R_2} & R_1 + R_3 + \frac{R_1 R_3}{R_2} \\ \frac{1}{R_2} & \frac{R_1}{R_2} + 1 \end{pmatrix}$$



$$T_{eq} = T_2 \times T_3 \times T_1$$

$$\begin{aligned}
 T_{eq} &= \begin{pmatrix} 1 & 0 \\ 1/R_2 & 1 \end{pmatrix} \begin{pmatrix} 1 & R_3 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/R_1 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 1 & R_3 \\ 1/R_2 & \frac{R_3}{R_2} + 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/R_1 & 1 \end{pmatrix} \\
 &= \begin{pmatrix} 1 + \frac{R_3}{R_1} & R_3 \\ \frac{1}{R_2} + \frac{R_3}{R_1 R_2} + \frac{1}{R_1} & \frac{R_3}{R_2} + 1 \end{pmatrix}
 \end{aligned}$$



les quatre equations qui permettent de determiner l'etat du reseau sont :

- l'equation donnee par la source :

$$V_1 = E - R_g I_1 \quad (1)$$

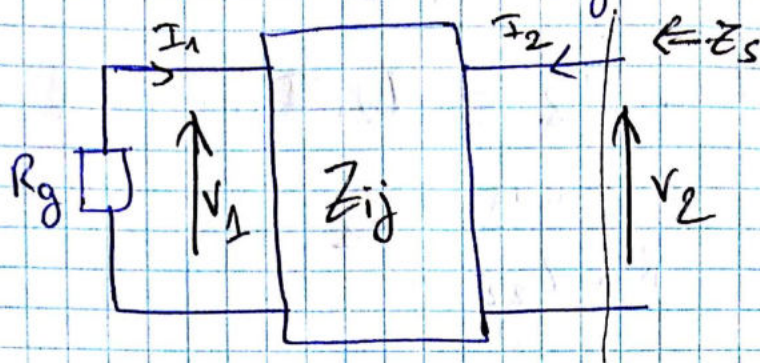
- l'equation donnee par la charge :

$$V_2 = -R I_2 \quad (2)$$

- le reseau :

$$\begin{cases}
 V_1 = Z_{11} I_1 + Z_{12} I_2 \\
 V_2 = Z_{21} I_1 + Z_{22} I_2
 \end{cases}$$

$Z_s = ??$  pour la déterminer il faut court-circuiter  $E$  et débrancher la charge en laissant  $R_g$ .



$$Z_s = \frac{V_2}{I_2} \Big|_{E=0} \quad \text{① après ①} \Rightarrow V_1 = -R_g I_1$$

on utilise l'équation ③, on obtient :

$$-R_g I_1 = Z_{11} I_1 + Z_{12} I_2$$

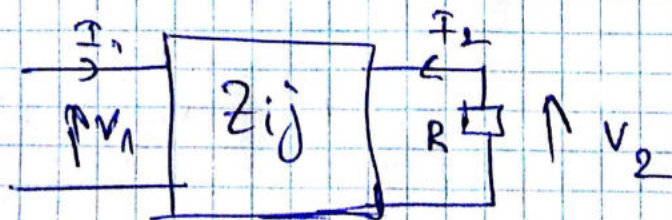
$$\Rightarrow -(R_g + Z_{11}) I_1 = Z_{12} I_2$$

$$\Rightarrow I_1 = \frac{-Z_{12}}{Z_{11} + R_g} I_2$$

Donc 
$$V_2 = \left( \frac{-Z_{21} Z_{12}}{Z_{11} + R_g} + Z_{22} \right) I_2$$

Donc 
$$Z_s = Z_{22} - \frac{Z_{21} Z_{12}}{Z_{11} + R_g}$$

$Z_E = ??$  on s'intéresse par à la source et on laisse  $R$ , (la charge).



ou a

$$-R I_2 = Z_{21} I_1 + Z_{22} I_2$$

$$\Rightarrow -(R + Z_{22}) I_2 = Z_{21} I_1$$

$$\Rightarrow I_2 = \frac{-Z_{21}}{R + Z_{22}} I_1$$

on remplace dans (3):

$$V_1 = Z_{11} I_1 - \frac{Z_{12} Z_{21}}{R + Z_{22}} I_1$$

$$V_1 = \left( Z_{11} - \frac{Z_{12} Z_{21}}{R + Z_{22}} \right) I_1$$

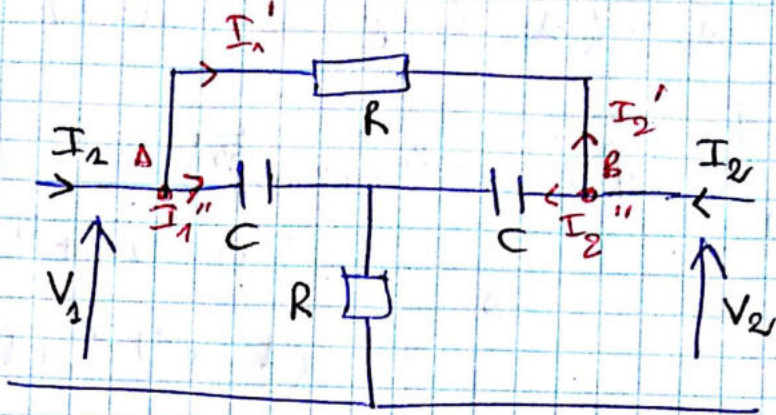
Donc  $Z_E = Z_{11} - \frac{Z_{12} Z_{21}}{R + Z_{22}}$

important Si la sortie n'est pas chargée

$R \rightarrow \infty$  donc  $Z_E = Z_{11}$



Ex (5)



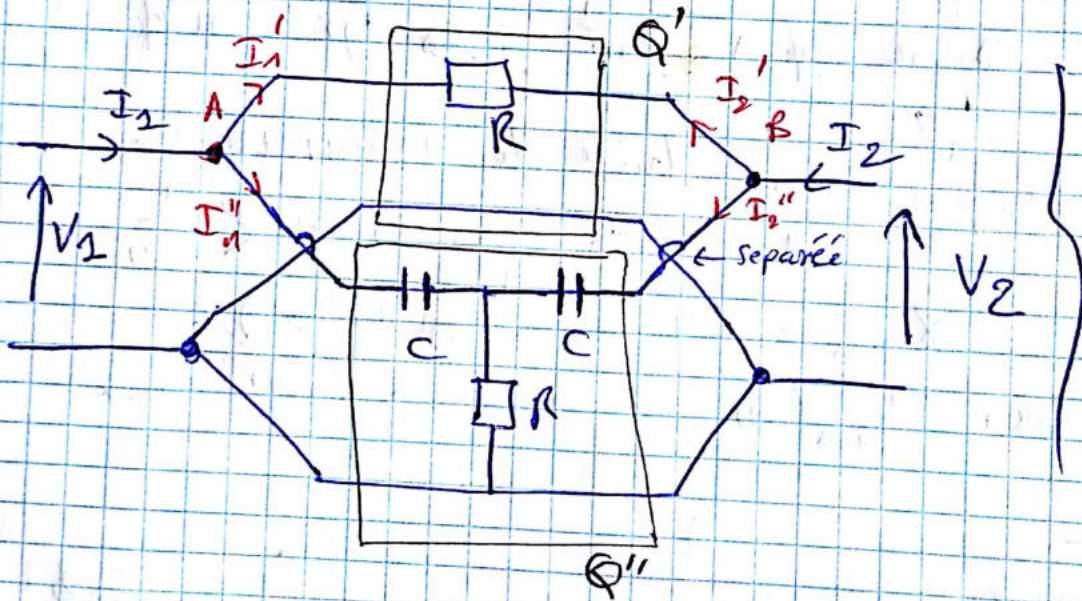
les éléments traversés par le m<sup>ê</sup>m courant entre les nœuds A et constituent un  $\hat{Q}$ .

$I_1'$  traverse R, donc R forme un  $\hat{Q}'$

$I_1''$  traverse C, R et C, donc CRC forme un  $\hat{Q}''$

On a

$$\begin{cases} I_1 = I_1' + I_1'' \\ I_2 = I_2' + I_2'' \end{cases}$$



en relie ces deux masses.

Cours | si  $m < 1$  (Régime oscillant)

$$T(\omega) = \frac{1}{\sqrt{(1-x^2)^2 + (2m x)^2}}$$

$$\frac{dT(\omega)}{dx} = 0 \Rightarrow \frac{-\frac{1}{2} \left( (1-x^2)^2 + (2m x)^2 \right)^{-\frac{1}{2}} \left( 2(1-x^2)(-2x) + 8m^2 x \right)}{\left( (1-x^2)^2 + (2m x)^2 \right)^{\frac{3}{2}}} = 0$$

$$\Rightarrow -\frac{1}{2} \left( 2(1-x^2)(-2x) + 8m^2 x \right) = 0$$

$$\Rightarrow -4x(1-x^2) + 8m^2 x = 0$$

$$\Rightarrow \boxed{-(1-x^2) + 2m^2 = 0}$$

$$\Rightarrow (1-x^2) = 2m^2$$

$$\Rightarrow x^2 = 1 - 2m^2$$

$$\Rightarrow \boxed{x = \sqrt{1 - 2m^2}} \quad \text{tjr positif (fréquence positive)}$$

la pulsation réduite de résonance.

$$1 - 2m^2 > 0 \Rightarrow m < \frac{1}{\sqrt{2}}$$

$$X_{\max} = \sqrt{1 - 2m^2}$$

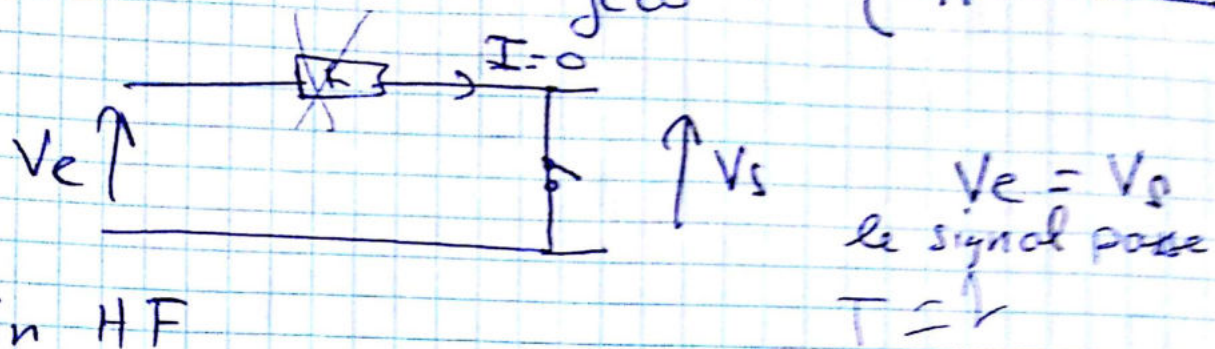
pour trouver  $T_{\max}(\omega)$  on remplace  ~~$T(\omega)$~~  par  $X_{\max}$

$$T_{\max} = \frac{1}{\sqrt{\left[ 1 - (1 - 2m^2) \right]^2 + \left( 4m \sqrt{1 - 2m^2} \right)^2}}$$

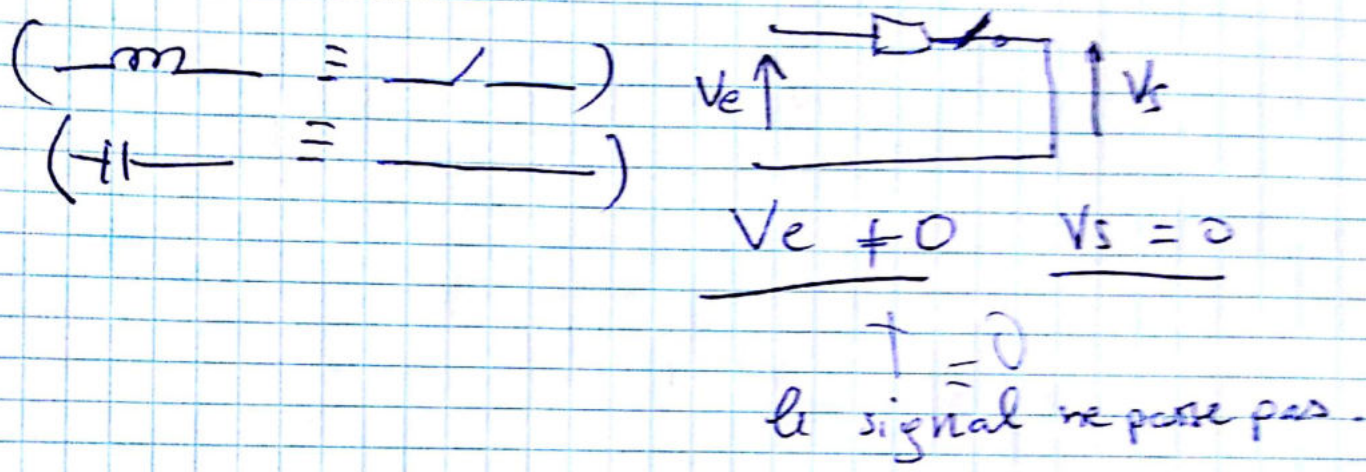
$$X_m = X_r = \frac{\omega_r}{\omega_0} \Rightarrow \omega_r = \omega_0 \sqrt{1 - 2m^2}$$

Exemple :

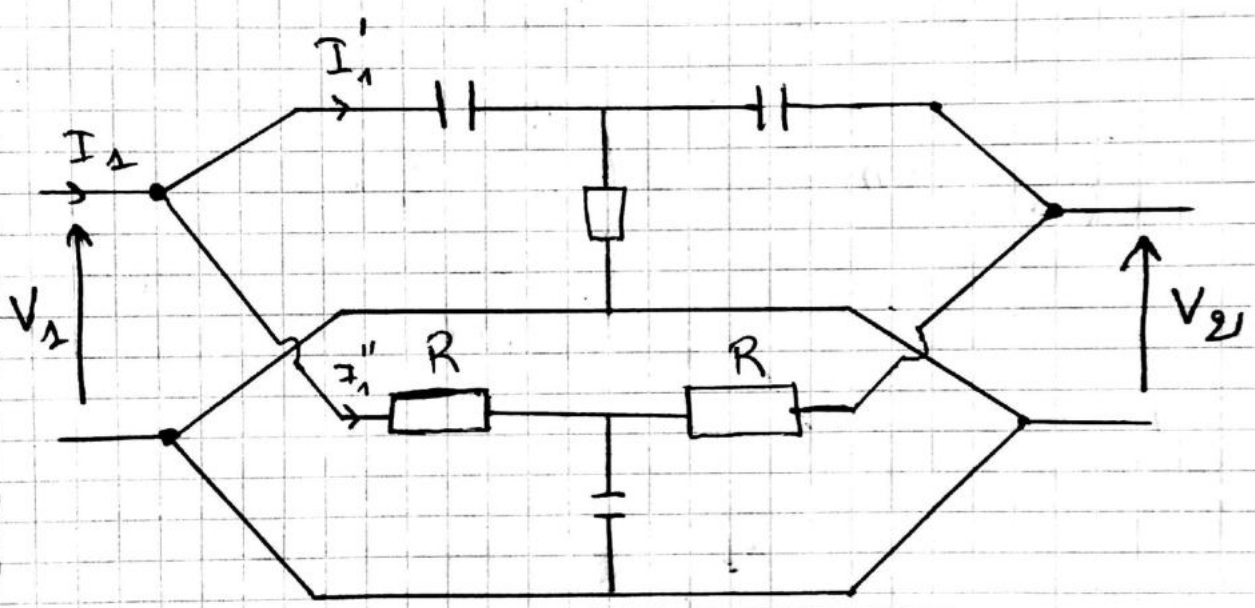
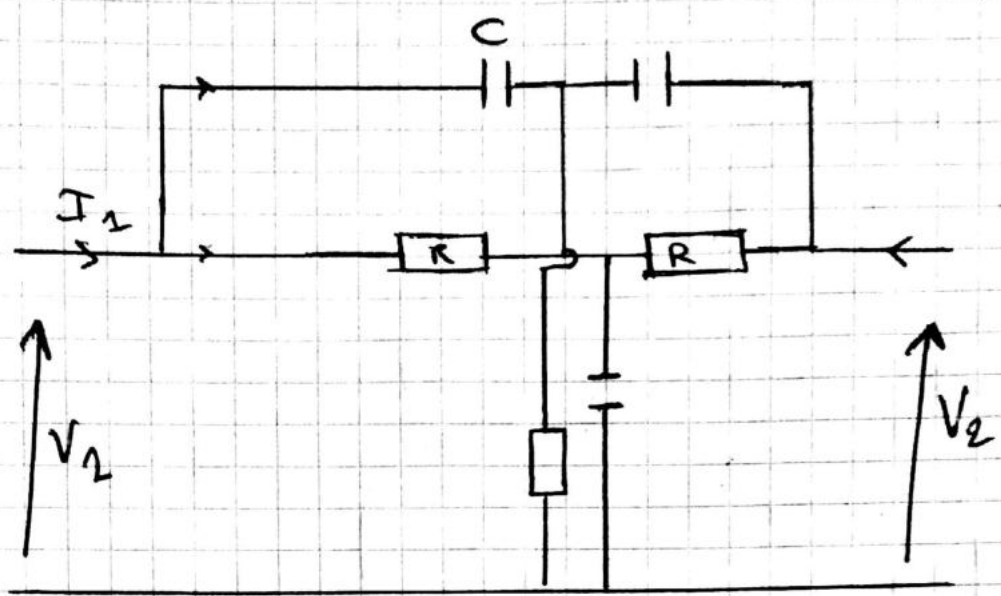
En BF  
 $\omega \rightarrow 0 \Rightarrow Z_L = jL\omega \rightarrow 0$  (—m— ≡ —)  
 $Z_C = \frac{1}{jC\omega} \rightarrow \infty$  (—|— ≡ —)



En HF  
 $\omega \rightarrow \infty$

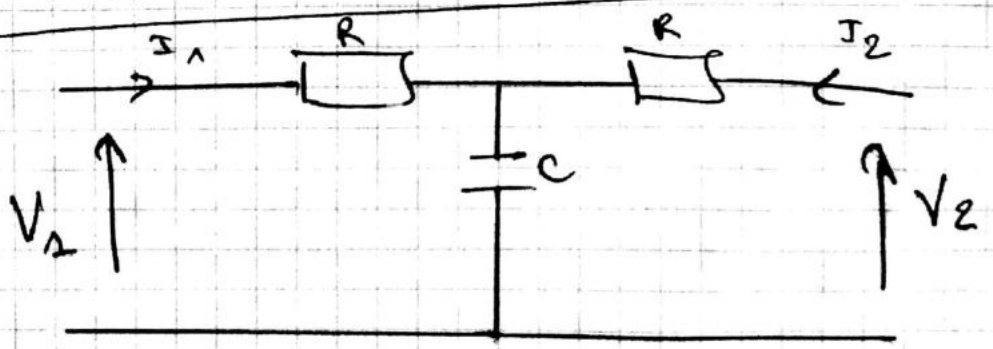


Suite Ex (5) Figure (7)



$$\text{com } Z = \begin{pmatrix} R + Z_c & -Z_c \\ -Z_c & R + Z_c \end{pmatrix}$$

$$(\text{com } Z)^t = \begin{pmatrix} R + Z_c & -Z_c \\ -Z_c & R + Z_c \end{pmatrix}$$



$$Z = \begin{pmatrix} R+Z_c & Z_c \\ Z_c & R+Z_c \end{pmatrix}$$

$$\det Z = (R+Z_c)^2 - Z_c^2$$

$$\det Z = R^2 + 2Z_c R$$

$$\text{com } Z = \begin{pmatrix} R+Z_c & -Z_c \\ -Z_c & R+Z_c \end{pmatrix}$$

$$(\text{com } Z)^t = \begin{pmatrix} R+Z_c & -Z_c \\ -Z_c & R+Z_c \end{pmatrix}$$

$$Y'' = \frac{1}{\det Z} (\text{com } Z)^t = \begin{pmatrix} \frac{R+Z_c}{R^2+2Z_c R} & \frac{-Z_c}{R^2+2Z_c R} \\ \frac{-Z_c}{R^2+2Z_c R} & \frac{R+Z_c}{R^2+2Z_c R} \end{pmatrix}$$

$$Y = Y' + Y''$$

$$Y' = \begin{pmatrix} \frac{Z_c+R}{Z_c^2+2Z_c R} & \frac{-R}{Z_c^2+2Z_c R} \\ \frac{-R}{Z_c^2+2Z_c R} & \frac{Z_c+R}{Z_c^2+2Z_c R} \end{pmatrix}$$

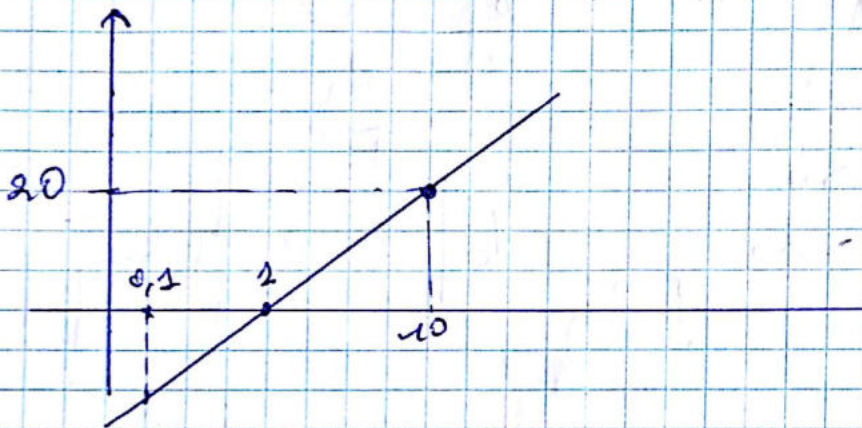
## Serie N°2:

### Exercice ①

$$T(j\omega) = jx = j \frac{\omega}{\omega_0}$$

$$|T(\omega)| = x$$

$$G_{dB} = +20 \log x \quad (+20 \text{ dB / décade})$$



la pulsation de coupure n'est pas définie

$$T(j\omega) = 1 + jx$$

$$|T(\omega)| = \sqrt{1 + x^2}$$

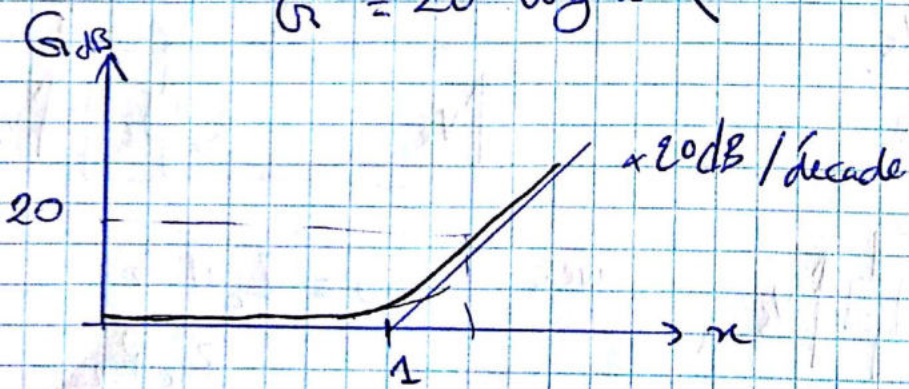
$$G_{dB} = 20 \log(\sqrt{1 + x^2})$$

en BF:  $x \ll 1$

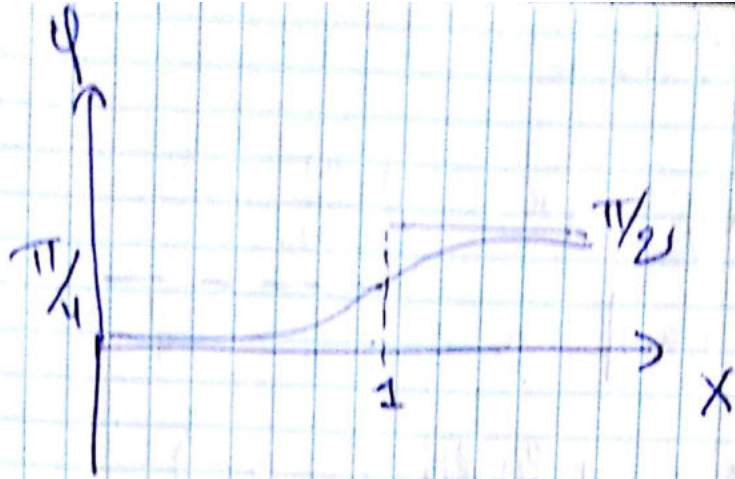
$$G_{dB} \longrightarrow 0$$

en HF:  $x \gg 1$

$$G = 20 \log x \quad (20 \text{ dB / décade})$$

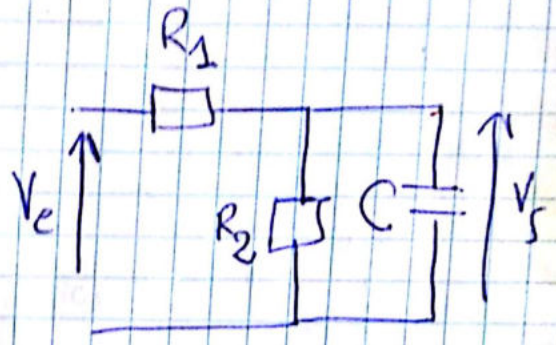
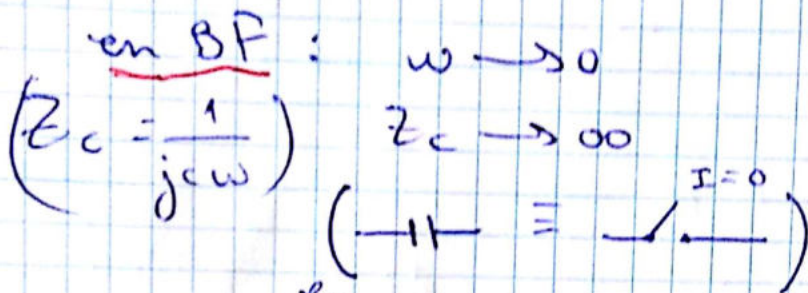


$$x = \frac{\omega}{\omega_0} = 1 \quad \Rightarrow \quad \omega = \omega_0$$

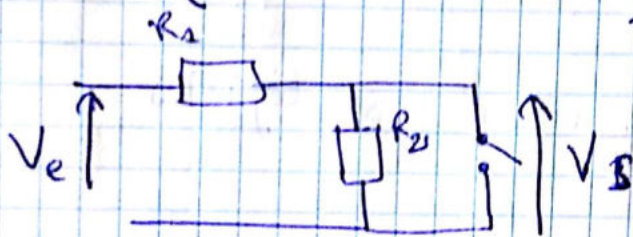


$$X=1 \rightarrow \varphi = \arg(1+j) = \arctg 1 = \frac{\pi}{2}$$

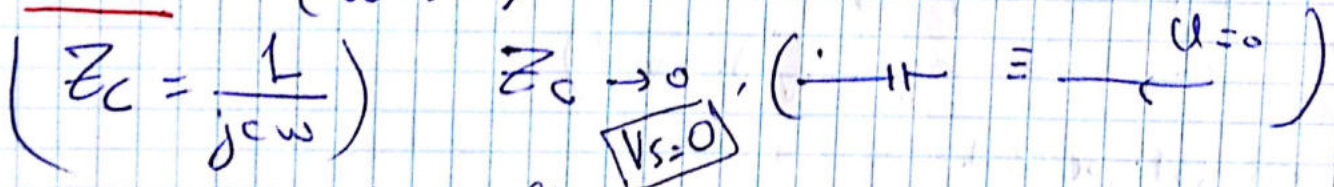
Ex (2)



$$V_s = \frac{R_2}{R_1 + R_2} V_e$$



en HF : ( $\omega \rightarrow \infty$ )

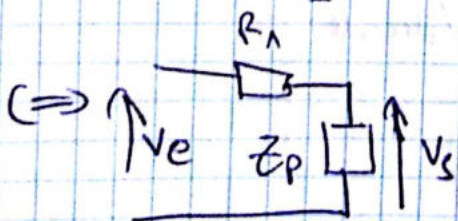


⇒ Il s'agit d'un filtre passe bas

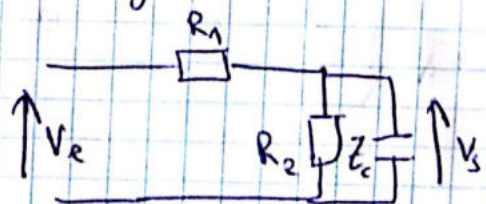
la pulsation de coupure n'est pas définie

(2)

$$T(j\omega) = \frac{V_s}{V_e}$$



avec



$$Z_p = Z_C \parallel R_2 = \frac{Z_C R_2}{Z_C + R_2}$$

diviseur de tension

$$\frac{V_s}{V_e} = \frac{Z_p}{Z_p + R_1} = T(j\omega)$$

$$\Rightarrow T(j\omega) = \frac{1}{1 + \frac{R_1}{Z_p}}$$

$$T(j\omega) = \frac{1}{1 + \frac{R_1(Z_c + R_2)}{Z_c \cdot R_2}}$$

$$\Rightarrow T(j\omega) = \frac{1}{1 + \frac{R_1}{R_2} + \frac{R_1}{Z_c}} = \frac{R_2}{R_2 + R_1 + j\omega R_1 R_2}$$

$$\Rightarrow T(j\omega) = \frac{R_2}{R_1 + R_2} \left( \frac{1}{1 + j\omega \frac{R_1 R_2}{R_1 + R_2}} \right)$$
$$= T_0 \frac{1}{1 + j\omega \tau}$$

avec  $T_0 = \frac{R_2}{R_1 + R_2}$  /  $\omega_0 = \frac{R_1 + R_2}{C R_1 R_2}$  /  $x = \frac{\omega}{\omega_0}$

$$|T(j\omega)| = T(\omega) = \frac{1}{2} \cdot \frac{1}{\sqrt{1+x^2}}$$

$$\bullet G_{dB} = 20 \log T(\omega) = -20 \log 2 - 20 \log \sqrt{1+x^2}$$

$$\bullet \varphi = \arg T(j\omega) = -\arg(1 + jx)$$

• les asymptotes:

En BF: ( $x \ll 1$ )

$$G_{dB} \longrightarrow -20 \log 2 = -6 \text{ dB}$$

$$\varphi \longrightarrow 0$$

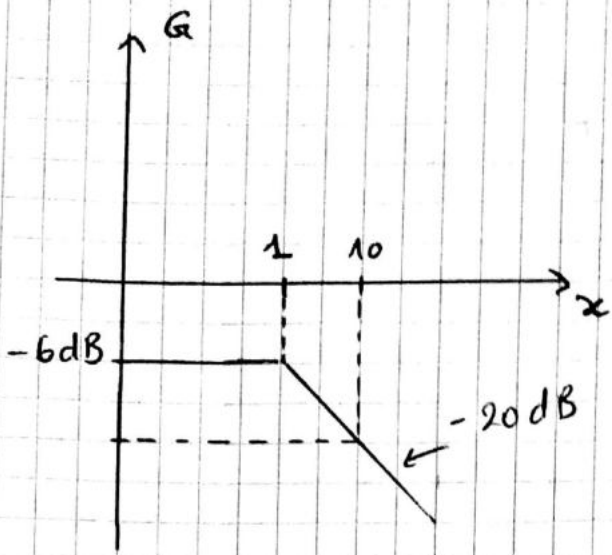
En HF ( $x \gg 1$ )

$$G_{dB} \longrightarrow -6 \text{ dB} - 20 \log x$$

$$\varphi = -\arg(jx)$$

$$\varphi \longrightarrow -\frac{\pi}{2}$$

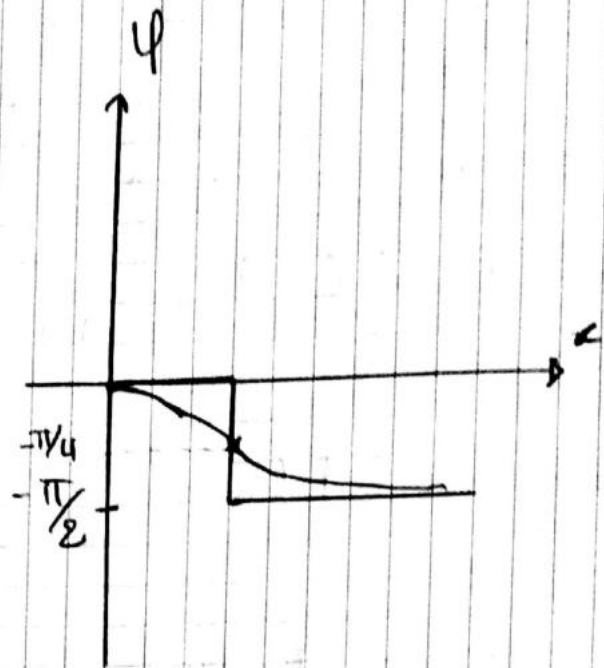




$$\frac{x=1}{-20 \log 2 - 20 \log \sqrt{2}} \approx -9 \text{ dB}$$

$$\bullet |T(\omega = \omega_c)| = \frac{T_{\max} = T_0}{\sqrt{2}}$$

↳ la bande passante  
 $[0, \omega_c]$



$$\Rightarrow \frac{1}{2} \cdot \frac{1}{\sqrt{1 + \left(\frac{\omega_c}{\omega_0}\right)^2}} = \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$$

$$\Rightarrow \sqrt{1 + \left(\frac{\omega_c}{\omega_0}\right)^2} = \sqrt{2}$$

$$\Rightarrow 1 + \left(\frac{\omega_c}{\omega_0}\right)^2 = 2$$

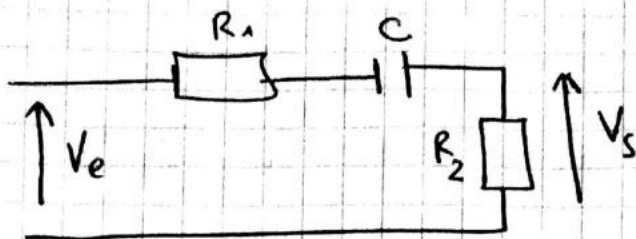
$$\Rightarrow \left(\frac{\omega_c}{\omega_0}\right)^2 = 1 \Rightarrow \frac{\omega_c}{\omega_0} = 1$$

$$\Rightarrow \boxed{\omega_c = \omega_0}$$

$$\rightarrow \boxed{\omega_c = \omega_0}$$

~~ω\_c = ω\_0~~  
~~ω\_c = ω\_0~~

Ex 3



① On a  $T(j\omega) = \frac{V_s}{V_e}$

le diviseur de tension  $\frac{V_s}{V_e} = \frac{R_2}{R_2 + Z_C + R_1}$

alors  $T(j\omega) = \frac{R_2}{R_2 + R_1 + \frac{1}{j\omega C}}$

$T(j\omega) = \frac{R_2 j\omega C}{1 + j\omega C(R_1 + R_2)} = \frac{jX_1}{1 + jX_2}$

$X_2 = C\omega(R_1 + R_2) = \frac{\omega}{\omega_0}$

$\omega_0 = \frac{1}{C(R_1 + R_2)}$

$X_1 = R_2 C \omega = \frac{\omega}{\omega_1}$

$\omega_1 = \frac{1}{R_2 C}$

$T(j\omega) = T_1 \times T_2$

$\begin{cases} T_1 = jX_1 \\ T_2 = \frac{1}{1 + jX_2} \end{cases}$

$T(\omega) = T_1(\omega) T_2(\omega)$

$G_{dB} = G_1 + G_2$

$G_{dB} = 20 \log X_1 - 20 \log \sqrt{1 + X_2^2}$

En BF:

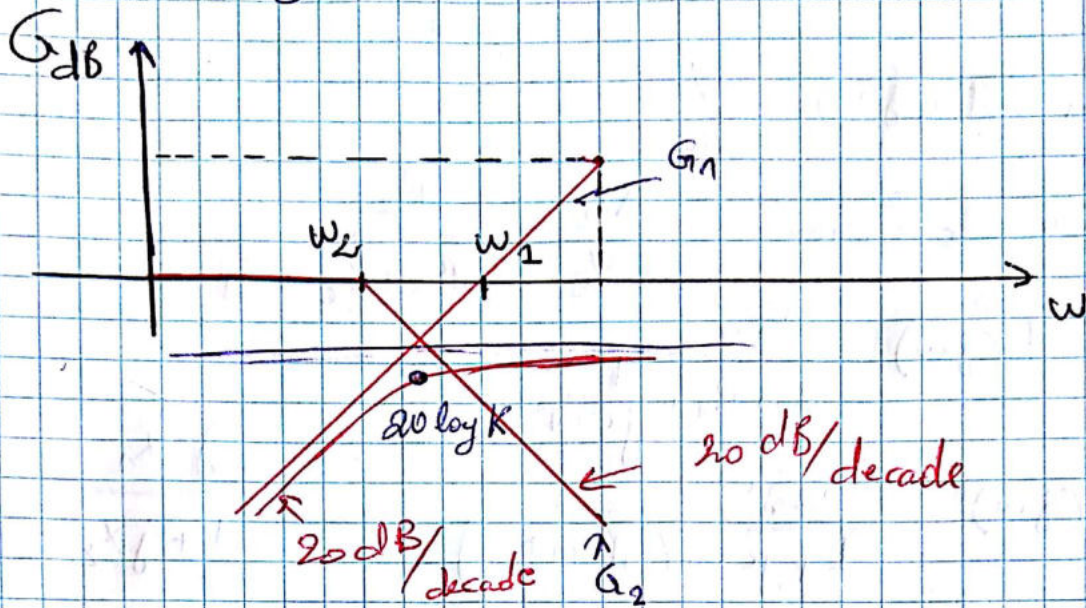
$G_1 \longrightarrow 20 \log X_1 \quad | \quad G_2 \longrightarrow 20 \log X_2$   
 $G_2 \longrightarrow 0$

En HF:

$G_1 \longrightarrow 20 \log X_1 \quad | \quad G_2 \approx 20 \log \frac{X_1}{X_2} = 20 \log \frac{\omega_1}{\omega_2}$   
 $G_2 \longrightarrow -20 \log X_2$   
 $\Rightarrow G \longrightarrow 20 \log K \leftarrow = 20 \log \left( \frac{R_2}{R_1 + R_2} \right)$

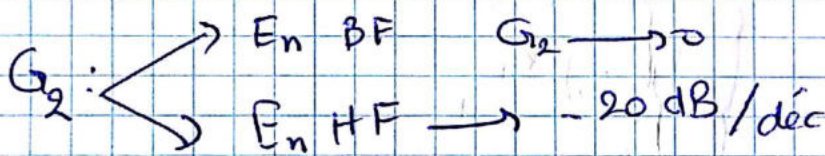
$\omega_2 < \omega < \omega_1$

$-20 \log x_2 + 20 \log x_1 \approx 20 \log K$



$G_{dB} = G_1 + G_2 = 20 \log \frac{\omega}{\omega_1} - 20 \log \sqrt{1 + (\frac{\omega}{\omega_0})^2}$

$G_1$  admet asymptote de 20 dB/dec



$T(j\omega) = \frac{j \frac{\omega}{\omega_1}}{1 + j \frac{\omega}{\omega_0}} = T_1 \times T_2$

$\omega_0 = \frac{1}{(R_1 + R_2)C}$

$\omega_1 = \frac{1}{R_2 C}$

$\omega_0 < \omega_1 \quad \left| \quad K = \frac{R_2}{R_1 + R_2} \right.$

$\omega_2 < \omega < \omega_1$

$-20 \log \frac{x_1}{x_2} = 20 \log K$

$\omega < \omega_2$

$G \rightarrow 20 \log x_1$

$\omega > \omega_1$

$20 \log x_1 - 20 \log x_2$

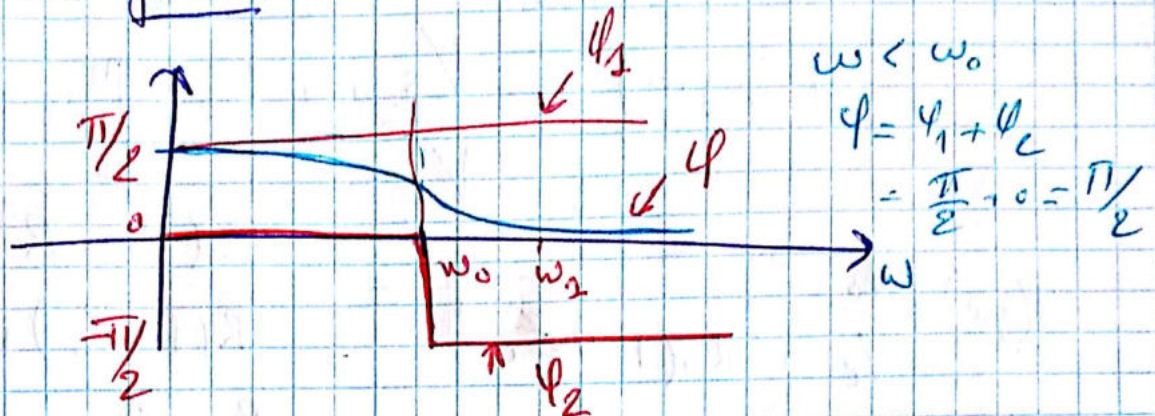
$$\omega_1 = \omega_2$$

$$G \approx 20 \log \frac{\omega}{\omega_1} - 20 \log \left( \sqrt{1 + \left( \frac{\omega}{\omega_0} \right)^2} \right)$$

$$\stackrel{\omega = \omega_2}{=} 20 \log \frac{\omega_2}{\omega_1} - 20 \log \sqrt{2}$$

$$G \approx 20 \log 2 - 3 \text{ dB}$$

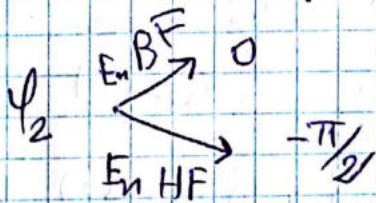
la courbe de phase



$$\varphi = \varphi_1 + \varphi_2$$

$$T_1 = j \frac{\omega}{\omega_1}$$

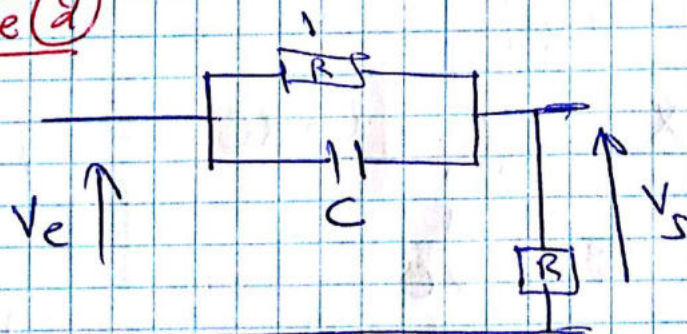
$$\varphi_1 \rightarrow \frac{\pi}{2}$$



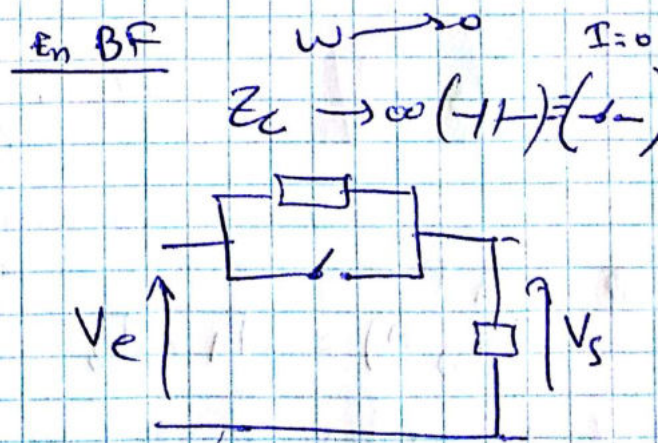
$$T(j\omega_0) = \frac{j \frac{\omega_0}{\omega_1}}{1 + j} = \arg(j \frac{\omega_0}{\omega_1}) - \arg(1 + j)$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

Figure 2



$$\frac{V_s}{V_e} = \frac{R}{2R} \Rightarrow V_s = \frac{1}{2} V_e$$



Ni passe bas ni passe haut / correction de phase  
 avancement de phase

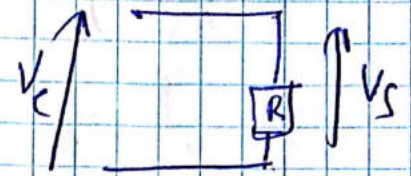
En HF

$$\omega \rightarrow \infty$$

$$Z_c \rightarrow 0$$

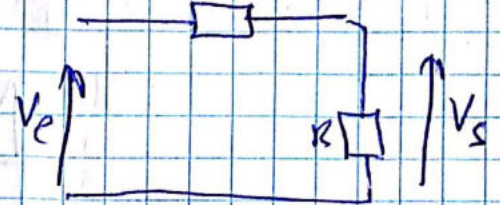
$$V_s = V_e$$

$$\left( \text{---} \parallel \text{---} \right) \equiv \left( \frac{V_I}{V=0} \right)$$



fonction de Transfert

$$T(j\omega) = \frac{V_s}{V_e}$$



$$T(j\omega) = \frac{V_s}{V_e} = \frac{R}{R + (R \parallel Z_c)} = \frac{R(Z_c + R)}{R(R + Z_c) + RZ_c} = \frac{R + Z_c}{R + Z_c + Z_c}$$

$$T(j\omega) = \frac{1 + jRC\omega}{2 + jRC\omega}$$

$$T(j\omega) = \frac{1 + jX}{2 + jX}$$

affaiblissement de  $\left[ \frac{1}{2} \right]$

$$\text{avec } \left. \begin{array}{l} X = \frac{\omega}{\omega_0} = \omega RC \\ \omega_0 = \frac{1}{RC} \end{array} \right\}$$

$$T(j\omega) = T_1 \times T_2 \quad \text{avec } \left. \begin{array}{l} T_1 = 1 + jX \\ T_2 = \frac{1}{2 + jX} \end{array} \right\}$$

$$T_1 = \sqrt{1+x^2} \rightarrow G_1 = 20 \log \sqrt{1+x^2}$$

$$T_2 = \frac{1}{\sqrt{4+x^2}} \rightarrow G_2 = -20 \log \sqrt{4+x^2}$$

$$G_{dB} = G_1 + G_2$$

En BF ( $x \ll 2$ )

$$G_1 \rightarrow 0$$

$$G_2 \rightarrow -20 \log 2 \approx -6 \text{ dB}$$

Donc

$$G \rightarrow -6 \text{ dB}$$

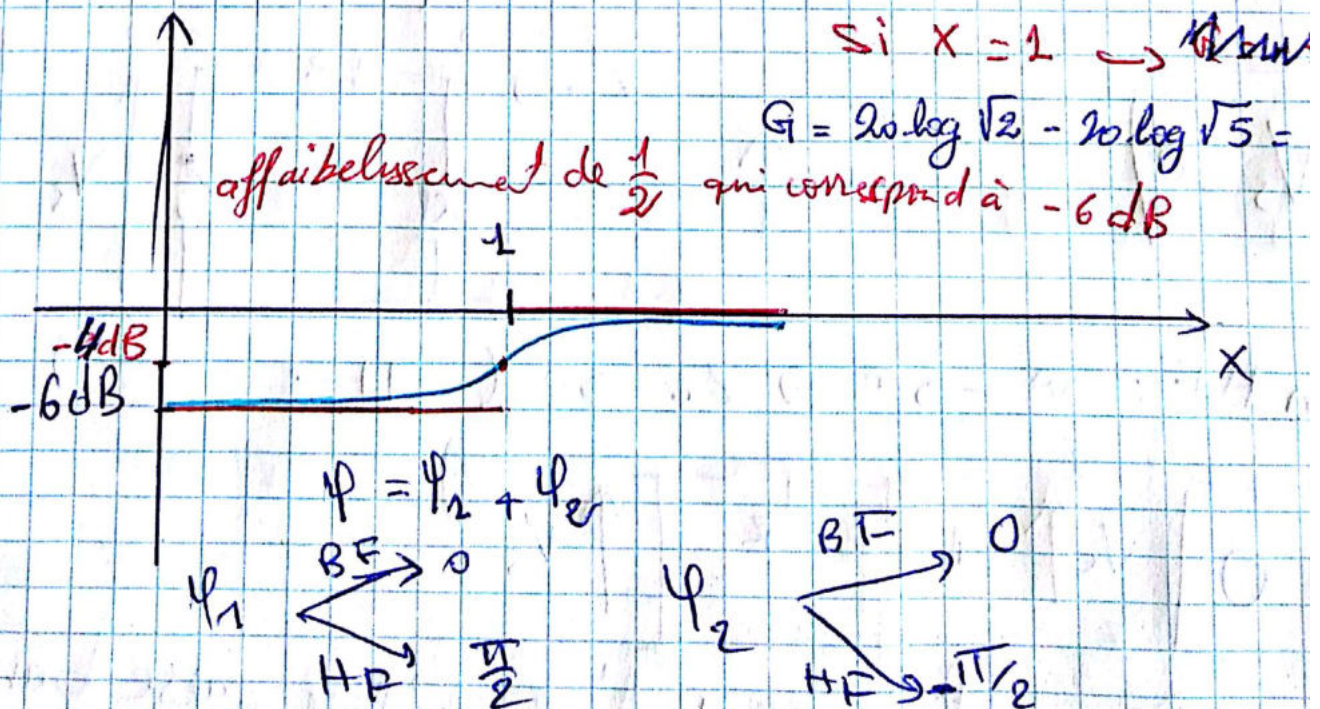
En HF

$$G_1 \rightarrow 20 \log x$$

$$G_2 \rightarrow -20 \log x$$

$$G \rightarrow 0$$

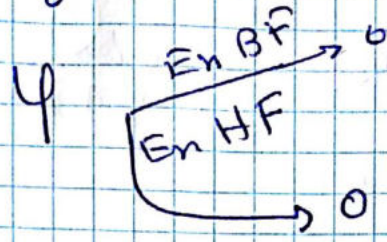
★ la courbe du Gain en dB admet : une droite de pente  $-6 \text{ dB}$  (en BF) et une droite de  $0 \text{ dB}$  (en HF).



$$T_2 = \frac{1}{2 + jx}$$

$$|\psi_2| = -\arg(2 + jx) / \arg(a + ib)$$

$$2 \arctan\left(\frac{b}{a}\right)$$



$$n = 2$$

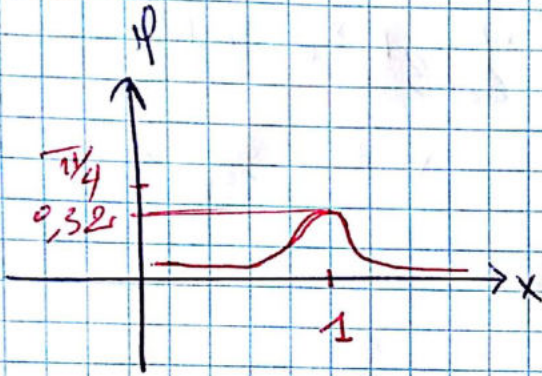
$$\phi_1 \approx \frac{\pi}{4}$$

$$\psi_2 = -\arg(2 + j)$$

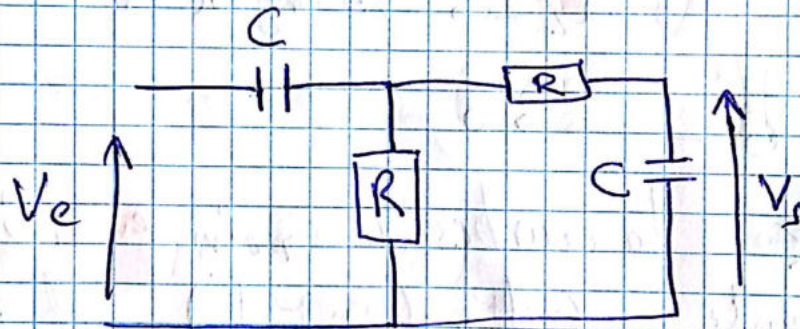
$$= -\arctg\left(\frac{1}{2}\right) = -26,56$$

$$\psi_2 = -0,46 \text{ rad}$$

$$\psi_2 = 0,32 \text{ rad}$$

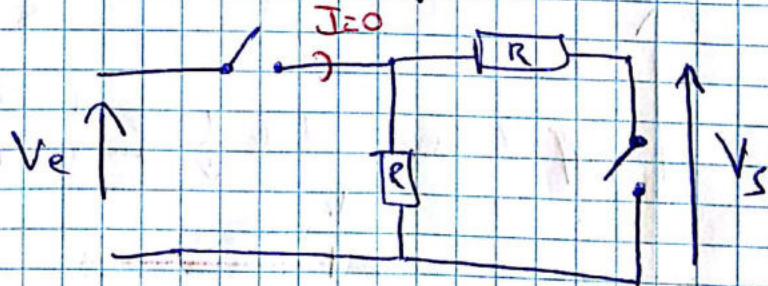


Ex (4)

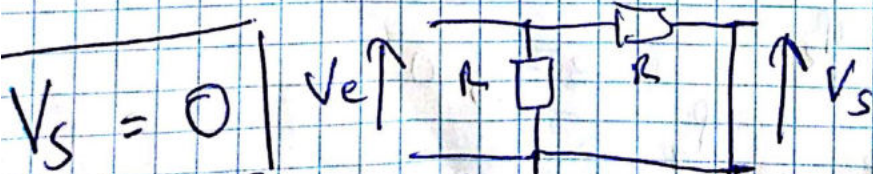


• En BF:  $\omega \rightarrow 0 \Rightarrow Z_C \rightarrow \infty$  ( — | — — — )

$$V_s = 0$$



• En HF:  $\omega \rightarrow \infty \Rightarrow Z_C \rightarrow 0$  ( — | — — — )



Donc il s'agit d'un filtre passe bande.

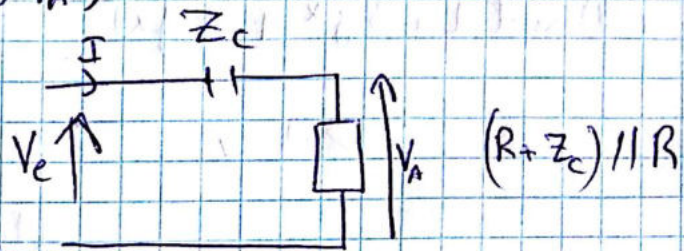
$$\textcircled{2} \quad \frac{V_s}{V_e} = \frac{V_s}{V_A} \cdot \frac{V_A}{V_e}$$

Diviseurs de tension de ( $V_s$  et  $V_A$ )

$$\frac{V_s}{V_A} = \frac{Z_c}{R + Z_c}$$

Diviseurs de tension de ( $V_e$  et  $V_A$ )

$$\frac{V_A}{V_e} = \frac{(R + Z_c) \parallel R}{(R + Z_c) \parallel R + Z_c}$$



$$\frac{V_A}{V_e} = \frac{\frac{(R + Z_c)R}{(R + Z_c) + R}}{\frac{(R + Z_c)R}{(R + Z_c) + R} + Z_c} \Rightarrow \frac{V_A}{V_e} = \frac{(R + Z_c)R}{(R + Z_c)R + Z_c(2R + Z_c)}$$

$$\Rightarrow \frac{V_A}{V_e} = \frac{(R + Z_c)R}{R^2 + Z_c^2 + 3RZ_c}$$

$$T(j\omega) = \frac{Z_c \cdot R}{R^2 + Z_c^2 + 3RZ_c} = \frac{Z_c R \cdot \frac{1}{Z_c^2}}{\frac{R^2 + Z_c^2 + 3RZ_c}{Z_c^2 Z_c^2 Z_c^2}}$$

Millmann

$$V_i = \frac{\sum \frac{V_i}{Z_i}}{\sum \frac{1}{Z_i}} \quad \text{avec } V_i \text{ potentiel}$$

Millmann

$$X = RC\omega = \frac{\omega}{\omega_0} \quad \left| \quad \omega_0 = \frac{1}{RC} \right.$$

$$2m = 3 \Rightarrow m = \frac{3}{2}$$



$$T(j\omega) = \frac{jR\omega}{1 + 3Rj\omega + (j\omega R)^2}$$

$$\Rightarrow T(j\omega) = \frac{jX}{1 + 3jX + (jX)^2}$$

$$\Rightarrow m = \frac{3}{2} \quad \text{et} \quad \omega_0 = \frac{1}{RC}$$

③ soit  $1 + 3jX + (jX)^2 \stackrel{u=jX}{=} 1 + 3u + u^2$

$$\Delta = 5 > 0, \quad u_1 = \frac{-3 - \sqrt{5}}{2}$$

$$u_2 = \frac{-3 + \sqrt{5}}{2}$$

le polynôme peut s'écrire

$$1 + 3u + u^2 = (u - u_1)(u - u_2)$$

et donc  $1 + 3jX + (jX)^2 = \left(jX + \frac{3 + \sqrt{5}}{2}\right) \left(jX + \frac{3 - \sqrt{5}}{2}\right)$

$$T(j\omega) = \frac{jX}{\left(jX + \frac{3 + \sqrt{5}}{2}\right) \left(jX + \frac{3 - \sqrt{5}}{2}\right)}$$

$$\Rightarrow T(j\omega) = \frac{jX}{\left(\frac{3 + \sqrt{5}}{2}\right) \left(\frac{3 - \sqrt{5}}{2}\right) \left(j \frac{X}{\frac{3 + \sqrt{5}}{2}} + 1\right) \left(j \frac{X}{\frac{3 - \sqrt{5}}{2}} + 1\right)}$$

$$\Rightarrow T(j\omega) = \frac{jX}{(1 + jX_1)(1 + jX_2)} = T_1 \times T_2 \times T_3$$

avec

$$\begin{cases} X_1 = \frac{X}{\frac{3 - \sqrt{5}}{2}} = \frac{\omega}{\omega_0 \left(\frac{3 - \sqrt{5}}{2}\right)} = \frac{\omega}{\omega_1} \\ X_2 = \frac{X}{\frac{3 + \sqrt{5}}{2}} = \frac{\omega}{\omega_0 \left(\frac{3 + \sqrt{5}}{2}\right)} = \frac{\omega}{\omega_2} \end{cases}$$

AN:

$$\text{avec } \begin{cases} \omega_1 = \left( \frac{3-\sqrt{5}}{2} \right) \omega_0 \approx 0,38\omega_0 \\ \omega_2 = \frac{3+\sqrt{5}}{2} \omega_0 \approx 2,61\omega_0 \end{cases}$$

$$\omega_1 < \omega_0 < \omega_2$$

$$4) T(j\omega) = jx \frac{1}{1+jx_1} \cdot \frac{1}{1+jx_2} = T_1 \times T_2 \times T_3$$

$$|T(j\omega)| = \frac{x}{\sqrt{1+x_1^2} \sqrt{1+x_2^2}}$$

$$G_{dB} = 20 \log |T(j\omega)| = 20 \log x - 20 \log \sqrt{1+x_1^2} - 20 \log \sqrt{1+x_2^2}$$

\*  $\omega < \omega_1 < \omega_2$   $\left( x_1 = \frac{\omega}{\omega_1} \ll 1 \text{ negligible devant } 1 \right)$

$$G_{dB} \Rightarrow 20 \log x$$

\*  $\omega_1 < \omega < \omega_2$

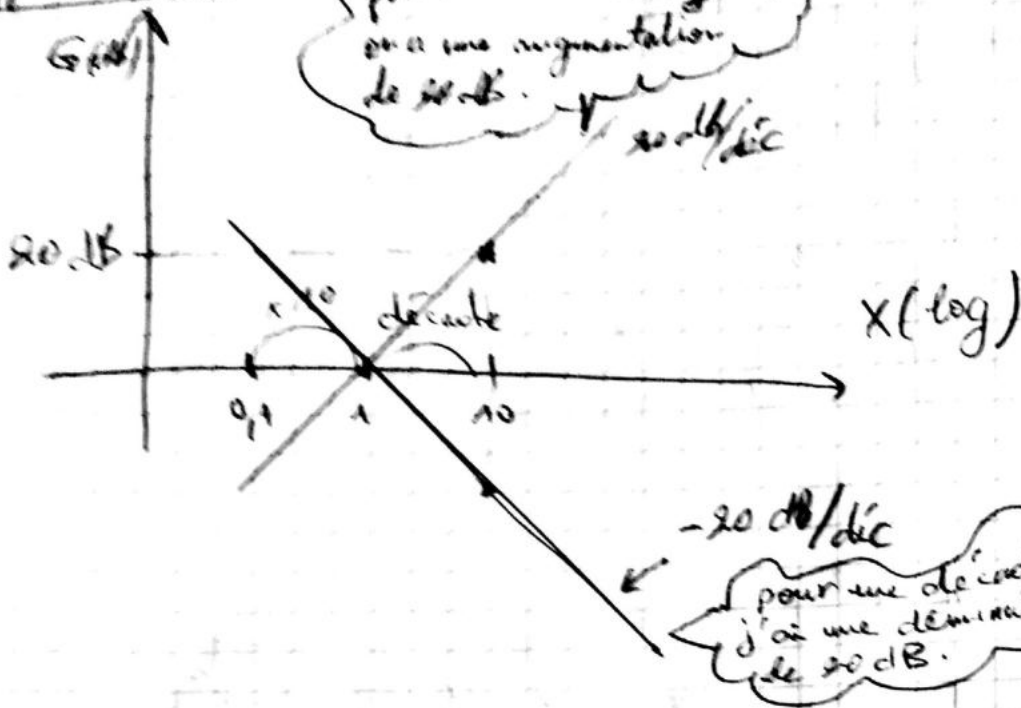
$$\begin{aligned} G_{dB} &\rightarrow 20 \log x - 20 \log x_1 = 20 \log \frac{\omega}{\omega_0} - 20 \log \frac{\omega}{\omega_1} \\ &= 20 \log \frac{\omega_1}{\omega_0} \\ &= 20 \log 0,38 \approx -8,4 \text{ dB} \end{aligned}$$

\*  $\omega > \omega_2$

$$\begin{aligned} G_{dB} &\rightarrow 20 \log x - 20 \log x_1 - 20 \log x_2 \\ &= -8,4 - 20 \log x_2 \end{aligned}$$

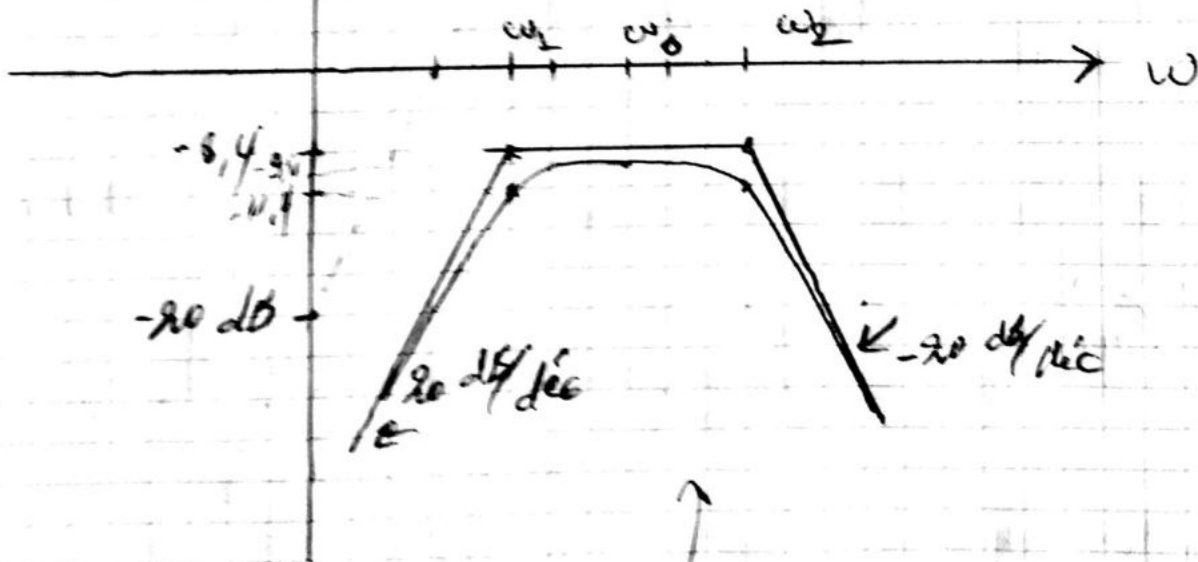
asymptote oblique à pente  $-20 \text{ dB/dec}$  passant par  $(\omega_2, -8,4 \text{ dB})$

Rappel pour



$$\omega_c = (\omega_1, \omega_2)$$

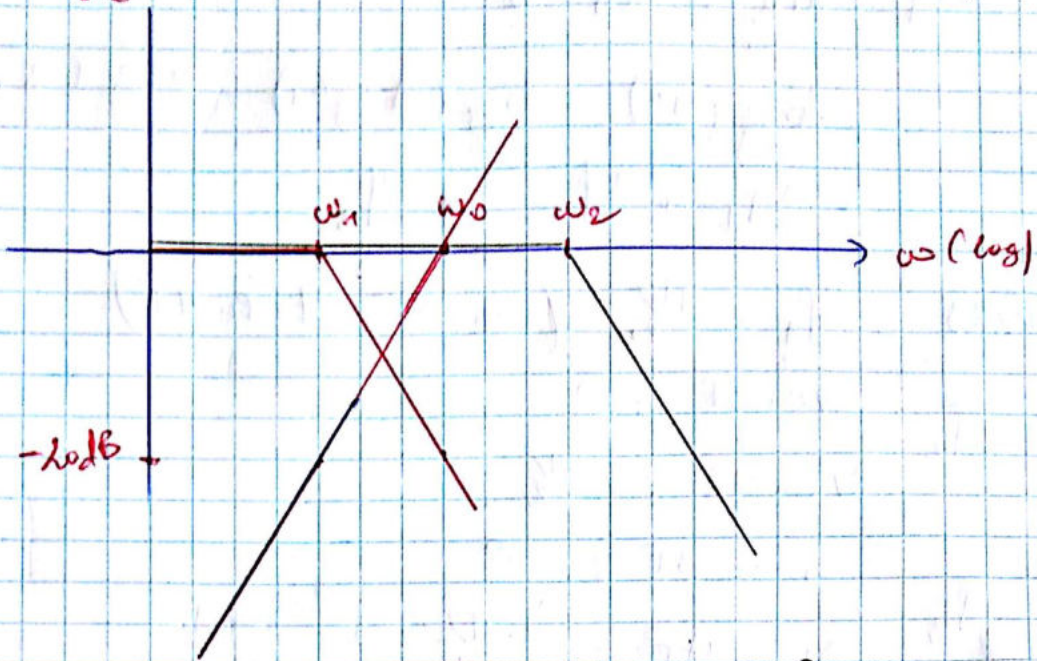
$$G(\omega = \omega_c) = G_{max} - 3 \text{ dB}$$



→ la courbe réel

$G(\text{dB})$

autre méthode en sépare ( $T_1, T_2, T_3$ )



on vérifie que  $G(\omega = \omega_c) = G_{\text{max}} - 3\text{dB}$

$$|T(j\omega)| = \frac{\omega/\omega_0}{\sqrt{1 + \left(\frac{\omega}{\omega_1}\right)^2} \sqrt{1 + \left(\frac{\omega}{\omega_2}\right)^2}}$$

$$|T_{\omega=\omega_1}| = \frac{\omega_1/\omega_0 = 0,99}{\sqrt{2} \sqrt{1 + \left(\frac{\omega_1}{\omega_2}\right)^2}} \approx 0,26$$

$$G_{\omega=\omega_2} = 20 \log 0,26 \approx -11,4 \text{ dB}$$

$$G_{\omega=\omega_2} \approx 20 \log 0,26 \approx -11,4 \text{ dB}$$

$$|T_{\omega=\omega_0}| = \frac{1}{\sqrt{1 + \left(\frac{\omega_0}{\omega_1}\right)^2} \sqrt{1 + \left(\frac{\omega_0}{\omega_2}\right)^2}}$$

$$G_{\text{dB}} \approx -9,58 \text{ dB}$$

4f)

$$\varphi = \arg(T(j\omega))$$

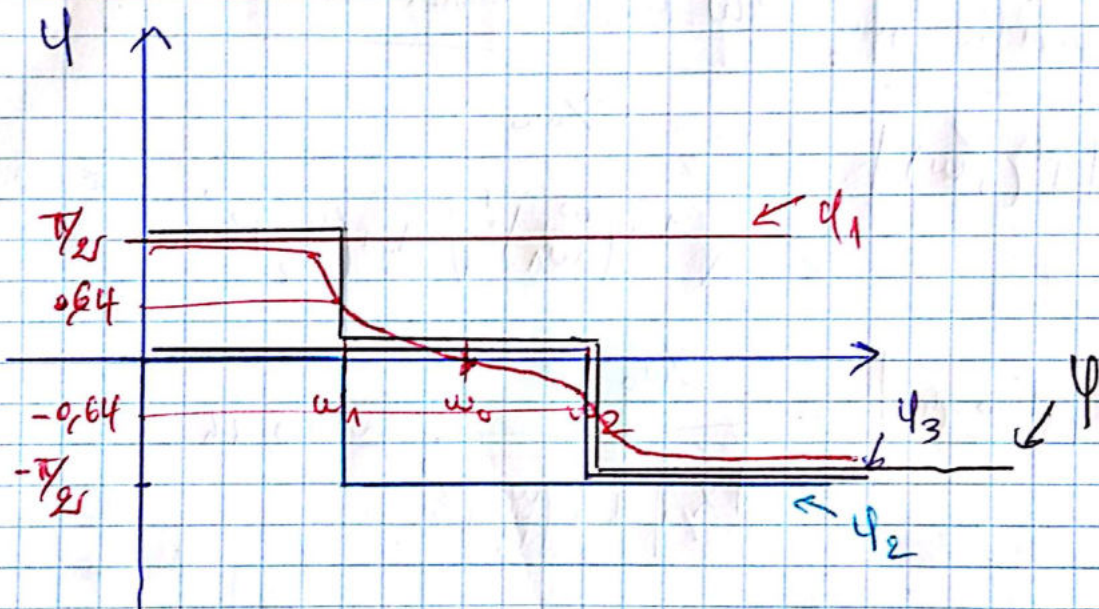
$$= \arg(j\kappa) - \arg(1+j\kappa_1) - \arg(1+j\frac{\omega}{\omega_2})$$

$$= \varphi_1 + \varphi_2 + \varphi_3$$

alors  $\varphi_1 = \frac{\pi}{2}$  (en BF et en HF)

$$\varphi_2 \begin{cases} \text{en BF} : 0 \\ \text{en HF} : -\frac{\pi}{2} \end{cases}$$

$$\varphi_3 \begin{cases} \text{en BF} : 0 \\ \text{en HF} : -\frac{\pi}{2} \end{cases}$$



on calcule  $\varphi_{\omega=\omega_2} = \arg(j \frac{\omega_1}{\omega_0}) - \arg(1+j) - \arg(1+j \frac{\omega_1}{\omega_2})$

$$= \frac{\pi}{2} - \frac{\pi}{4} - \arctg(\frac{\omega_1}{\omega_2})$$

$$\approx 0,64 \text{ rad}$$

$$\varphi_{\omega=\omega_2} = \arg(j \frac{\omega_2}{\omega_0}) - \arctg(\frac{\omega_2}{\omega_1}) - \frac{\pi}{4}$$

$$\approx -0,64 \text{ rad}$$

$$\varphi_{\omega=\omega_0} = \arg(j) - \arg(1+j \frac{\omega_0}{\omega_1}) - \arg(1+j \frac{\omega_0}{\omega_2})$$

$$\varphi_{\omega=\omega_0} = \frac{\pi}{2} - \operatorname{arctg}\left(\frac{\omega_0}{\omega_1}\right) - \operatorname{arctg}\left(\frac{\omega_0}{\omega_2}\right)$$

$$\varphi_{\omega=\omega_0} = 0$$